Exam
SOLID MECHANICS (NASM)
November 3, 2021, 18:45-21:45 h

This exam comprises four problems, for which one can obtain the following points:

Question	# points
1	$0.5 + 1 = 1.5$
2	$1 + 1.5 + 2 = 4.5$
3	$2.5 + 2.5 = 5$
4	2

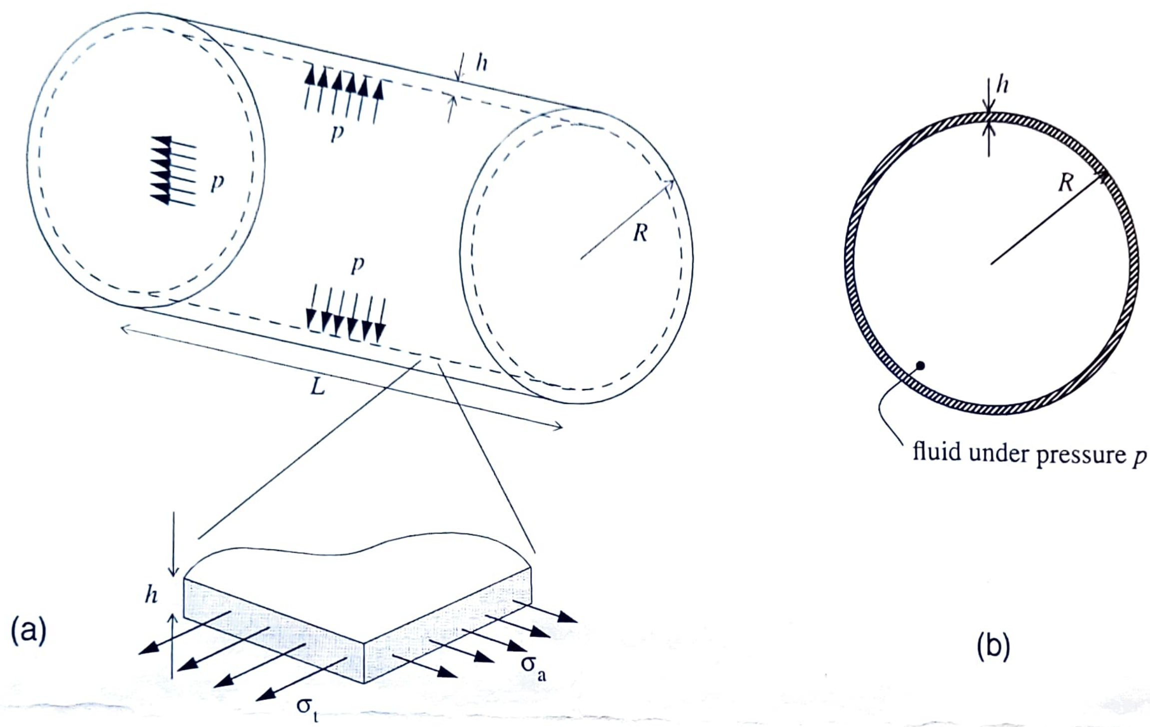
The number of points is indicated next to each subquestion inside a rectangular box in the right-hand margin on the next pages.

The exam grade is calculated as $9 * (\#\,\text{points})/13 + 1$. The final grade for this course is also based on the grade for the ComSol report.

Question 1 In tutorial exercise III.3 you have applied the principle of virtual work to show that the axial stress σ_a and the tangential stress σ_t in a thin-walled cylinder (radius R, wall thickness $h \ll R$) under an internal pressure p are given by

$$\sigma_t = 2\sigma_a = \frac{pR}{h}. \tag{1}$$

Alternatively, one may derive these expressions directly by equilibrium considerations of such a cylinder filled with a fluid, as illustrated in the axial view in figure (b) below. Note that it is assumed that p is so large that the influence of gravity can be ignored.



a. Before embarking on the equilibrium considerations, however, it may be good to think about the state of stress in the fluid. When the fluid does not flow, its response is elastic, yet with a shear modulus μ that is much smaller than the bulk modulus k. Show that in the limit $\mu/k \to 0$, the pressure distribution in the fluid is uniform. $\boxed{0.5}$

b. Knowing this, choose convenient sections of the system pipe+fluid to obtain the expressions (1) by consideration of equilibrium. Include clear sketches of the free-body diagrams you use. $\boxed{1}$

Question 2

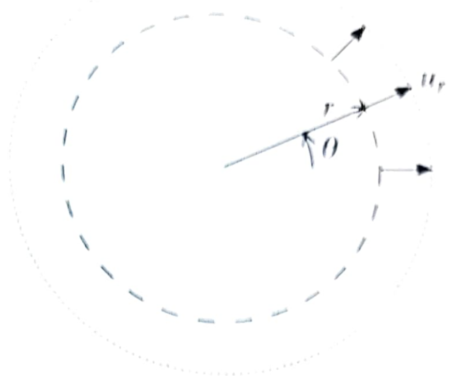
The relationship

$$\boldsymbol{\varepsilon} = \frac{1}{2}[\text{grad}\, \boldsymbol{u} + (\text{grad}\, \boldsymbol{u})^T]$$

for strain in terms of the displacement field is completely general (within the *geometrically linear theory*). In terms of Cartesian coordinates, with corresponding orthonormal base vectors $\boldsymbol{e}_i$, the corresponding expression for components simply reads

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i})$$

as used throughout this course. For other coordinate systems, such as polar coordinates, the expressions for the components can become somewhat cumbersome because the gradient for non-Cartesian coordinates is more involved.



Here we focus on the planar strain components in polar coordinates for the *special case of axisymmetric deformations*, as illustrated above. The latter means that the displacement field only has a component u_r in the radial direction (i.e., $u_\theta \equiv 0$) and that there is no dependence on the angle θ (or on the coordinate perpendicular to the plane). For such axisymmetric planar conditions, the in-plane shear component vanishes and the normal strain components are given by:

$$\varepsilon_{rr} = \frac{du_r}{dr}, \qquad \varepsilon_{\theta\theta} = \frac{u_r}{r}. \tag{2}$$

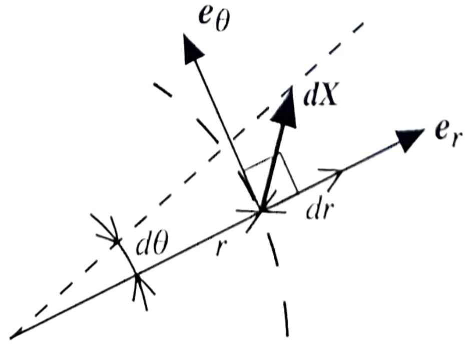
The objective of this problem is to derive these results.

a. Use symmetry to show that the shear component $\varepsilon_{r\theta} = 0$. $\boxed{1}$

b. For the determination of the normal strain components, let us return to the original definition of the strain tensor $\boldsymbol{\varepsilon}$ through

$$\boldsymbol{dx} \cdot \boldsymbol{dx} - \boldsymbol{dX} \cdot \boldsymbol{dX} = 2\boldsymbol{dX} \cdot \boldsymbol{\varepsilon} \cdot \boldsymbol{dX} \tag{3}$$

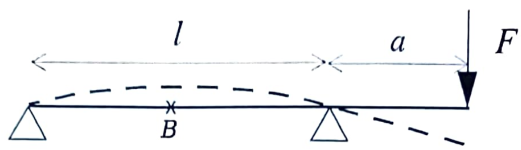
where $\boldsymbol{dX}$ and $\boldsymbol{dx}$ are vectors that characterize a line element in the reference and in the deformed configuration, respectively.



At an arbitrary radius r, choose $\boldsymbol{dX}$ according to $\boldsymbol{dX} = dr\boldsymbol{e}_r + rd\theta\boldsymbol{e}_\theta$, as illustrated above (note that $\boldsymbol{e}_r$ and $\boldsymbol{e}_\theta$ are orthonormal base vectors). Express $\boldsymbol{dx}$ in terms of the quantities introduced and the radial displacement u_r. 1.5

c. Derive the expressions (2) for the normal strain components. 2

Question 3 A beam with modulus E, moment of inertia I and length $l+a$ is in a 'diving board' configuration; that is, it is simply supported at two points (where it can freely rotate) and is loaded by a force F at the end, as shown below.



The aim now is to determine the deflection w at point B in the center of the two supports in terms of the specified variables.

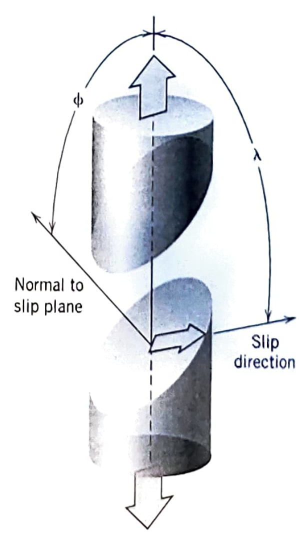
a. Let's do so by solving the differential equation (3.49) for beam bending through the following steps:

- determine the distribution of the bending moment $M(x)$ (wherever needed); 1
- specify the boundary conditions; 0.5
- solve Eq. (3.49). 1

b. An alternative approach is to focus on the section of length l between the two supports, and to:

- consider it as a free-body diagram including forces and torques; 0.5
- split this asymmetric problem into a symmetric and an anti-symmetric (or centro-symmetric) problem; 1
- use the Forget-Me-Nots to find the deflection at point B. 1

Question 4 The figure on the right is copied from the section on slip in single crystals from Callister's textbook (fifth edition) used in your Materials Science course. In a crystal subjected to uniaxial tension, as indicated in the figure by the vertical arrows, a slip system is identified by the angle ϕ between slip plane normal and tensile direction, and by the angle λ between slip direction and tensile axis.



Derive the relation between the applied stress σ and the shear stress resolved on this slip system, τ_R. 2